Intelligence 62 (2017) 54–70

Contents lists available at ScienceDirect

Intelligence

journal homepage: www.elsevier.com/locate/intell

Estimating the dimensionality of intelligence like data using Exploratory Graph Analysis

Hudson F. Golino [a], Andreas Demetriou [b,*]

[a] Universidade Salgado de Oliveira, Brazil
[b] University of Nicosia, Cyprus

ARTICLE INFO

Article history:
Received 25 July 2016
Received in revised form 6 January 2017
Accepted 20 February 2017
Available online 7 March 2017

ABSTRACT

This study compared various exploratory and confirmatory factor methods for recovering factors of cognitive test-like data. We first note the problems encountered by several widely used methods, such as parallel analysis, minimum average partial procedure, and confirmatory factor analysis, in estimating the number of dimensions underlying performance on test batteries. We then argue that a new method, Exploratory Graph Analysis (EGA), can more accurately uncover underlying dimensions or factors and demonstrate how this method outperforms the other methods. We use several published data sets to demonstrate the advantages of EGA. We conclude that a combination of EGA and confirmatory factor analysis or structural equation modeling may be the ideal in precisely specifying latent factors and their relations.

1. Introduction

Discovering dimensions (or factors) underlying human behavior and cognitive ability is central in psychology and the cognitive sciences. Factor analysis was developed to uncover the dimensions underlying a large number of measures of the behaviors or abilities of interest (Spearman, 1904; Carroll, 1993; Jensen, 1998). However, there is no agreement yet about the method of choice for identifying the best number of dimensions and their relations under various conditions of measurement, sampling of persons, and between dimensions' relations. Principal component analysis, factor analysis of various types and rotations, and, recently confirmatory factor analysis and structural equation modeling were advanced to cope with these problems.

Recently, Keith, Caemmerer and Reynolds (2016) showed that both parallel analysis (PA) and minimum average partial procedure (MAP) underestimate the number of dimensions in many realistic data condition, especially when the correlation between factors are high (.70) and the number of indicators per factor are low. Their results align with earlier research showing that both PA and MAP work well when there is a low or moderate correlation between factors, when the sample size is equal to or >500 and when the factor loadings are from moderate to high (Buja & Eyuboglu, 1992; Crawford et al., 2010; Garrido, Abad & Ponsoda, 2011; Green, Redell, Thompson & Levy, 2016; Timmerman & Lorenzo-Seva, 2011; Velicer, Eaton, & Fava, 2000, Velicer, 1976; Zwick & Velicer, 1986).

Simulation studies point to a relevant issue: PA and MAP fail to uncover the correct number of factors in situations approaching real intelligence datasets. Keith et al. (2016) suggested that researchers must use confirmatory factor analysis (CFA) guided by a relevant theory, because CFA was more accurate than other methods in recovering the correct number of dimensions in their simulation study. To deal with the fact that exploratory techniques did not correctly recover the number of factors in realistic data conditions, these authors strongly suggested using theory to guide the analysis.

Although useful when available, theory, in principle, may suggest but, cannot specify either the true number of dimensions in an instrument or how items in the instrument may relate to these dimensions. Obviously, there may be factors in the battery that are ignored or overlooked by the theory. Thus, CFA may be used to test if the theoretically expected dimensions are present in the data. However, when a CFA theory-based model fails to fit empirical data, other tools are needed to explore the structure of the instrument as precisely as possible.

We argued above that PA, MAP and other traditional techniques are not robust enough to estimate the number of factors underlying a given instrument, when the correlation between factors is high and the number of variables per factor is low. Thus, new robust techniques are needed. This paper presents a new method, Exploratory Graph Analysis (EGA; Golino & Epskamp, 2016), that is more powerful than earlier methods to estimate the number of dimensions in intelligence-like data. EGA was shown to outperform PA and MAP in conditions where these methods are not accurate: That is, when correlations between factors are high and the number of items per factor is low (Golino & Epskamp, 2016). Specifically, EGA is better in estimating the number of factors in situations which (1) are very close to what we find in real

* Corresponding author.
E-mail addresses: hfgolino@gmail.com (H.F. Golino), ademetriou@ucy.ac.cy (A. Demetriou).

http://dx.doi.org/10.1016/j.intell.2017.02.007
0160-2896/

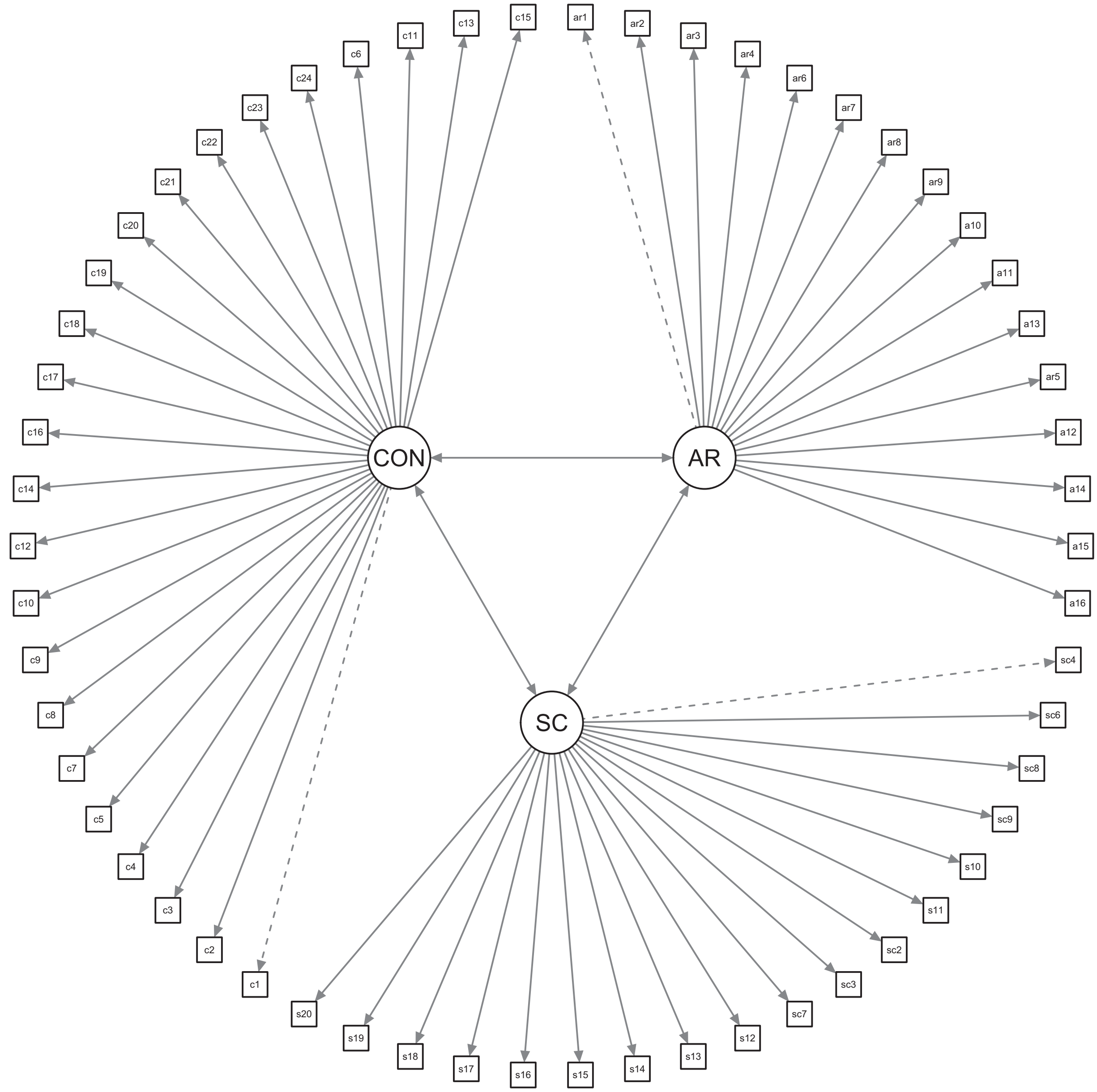

Fig. 1. The theoretical structure of the NIT subtests used in the current study. AR = arithmetical reasoning; SC = sentence completion; CON = concepts.

intelligence datasets but (2) traditional techniques underestimate the dimensions involved.

This paper is organized as follows. In the first section, EGA is briefly presented as part of a new area called *network psychometrics* (Epskamp, Maris, Waldorp & Borsboom, 2017). In the second section, EGA is applied on the datasets simulated by Keith et al., where PA and MAP presented low accuracy (i.e., high factor correlations, low factor loadings, and $N = 500$). Finally, in the third section, EGA is applied in three previously published datasets (Must & Must, 2013, 2014; Demetriou & Kazi, 2006; Žebec, Demetriou, & Kotrla-Topić, 2015) to show how this new technique can guide researchers in their search for the underlying dimensionality of intelligence like data.

1.1. Exploratory graph analysis: a brief overview

Exploratory Graph Analysis is part of a new area called network psychometrics (see Epskamp et al., 2017), which focuses on the estimation of undirected network models (i.e. Lauritzen, 1996a, b) to psychological datasets. This area has been applied in different areas of psychology, including psychopathology (e.g., Borsboom et al., 2011; Borsboom & Cramer, 2013; Fried et al., 2015) and developmental psychology (Kossakowski et al., 2015; van der Maas et al., 2006). In network psychometrics, the nodes represent psychological variables (e.g., test and/or questionnaire items, psychopathological symptoms, etc.) and the connection between nodes (i.e., edges) represents statistical relationships to be estimated (Epskamp & Fried, 2016). Thus, there is a fundamental distinction between network psychometrics and other types of network models, in which the links between nodes do not need to be estimated, such as social networks analysis (Epskamp & Fried, 2016). When analyzing data generated by psychological instruments, one may want to know if nodes are connected with each other, forming clusters standing for underlying latent variables. If a latent variable model is the true underlying causal model, we would expect indicators in a network model to form strongly connected clusters for each latent variable. Network models may be shown to be mathematically equivalent under certain conditions to latent variable models in both binary (Epskamp et al., 2017) and Gaussian datasets (Chandrasekaran, Parrilo & Willsky, 2010).

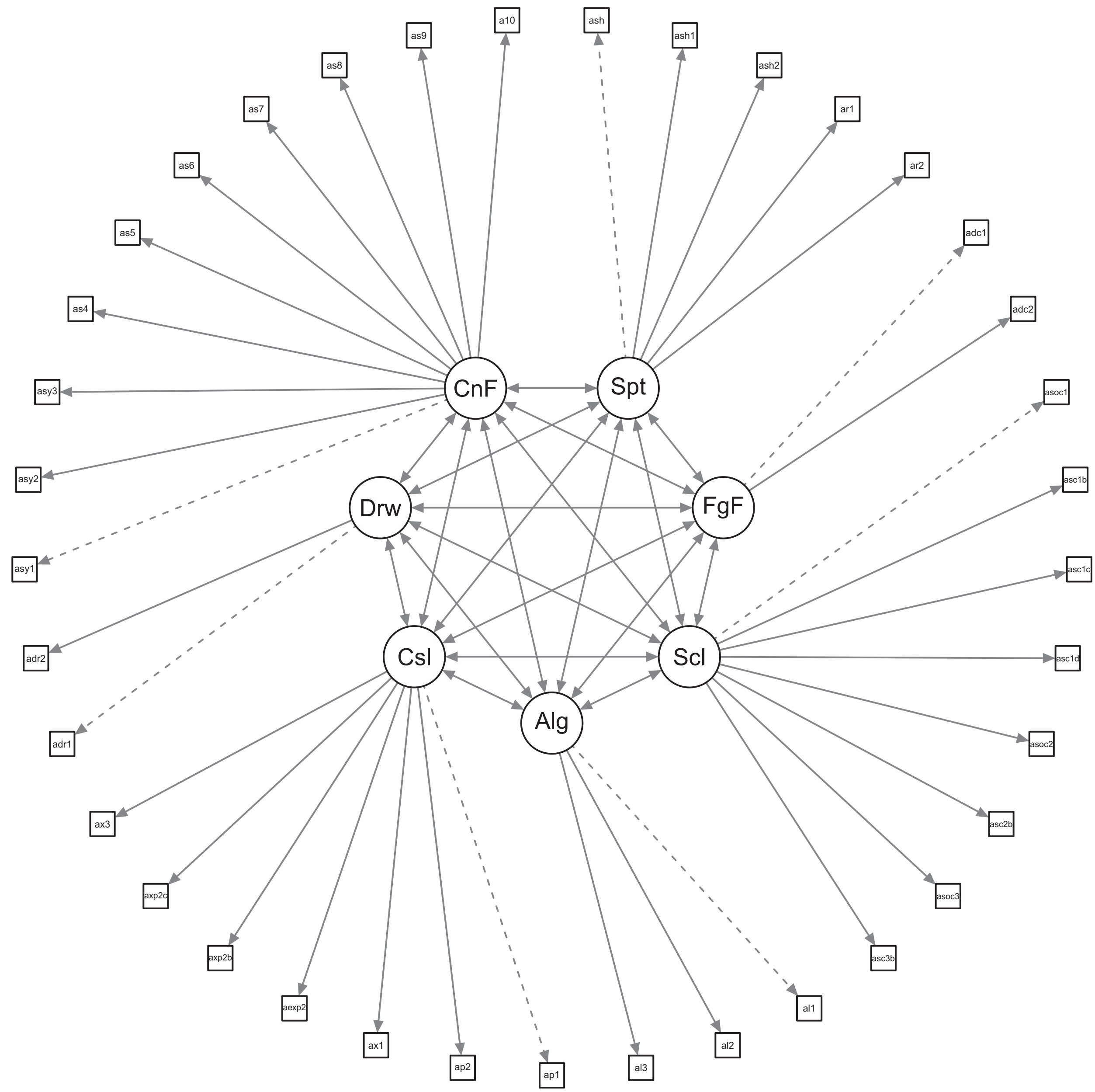

Fig. 2. The theoretical structure of the Demetriou and Kazi's (2006) instruments. Spt = spatial ability; FgF = figural fluency; Scl = social reasoning; Alg = algebraic reasoning; Csl = causal reasoning; Drw = drawing ability; CnF = conceptual fluency.

By defining a cluster as a group of connected nodes regardless of edge weight, Golino and Epskamp (2016) pointed to a fundamental rule of network psychometrics: Clusters in network equal latent variables. This is both an empirical finding (Cramer et al., 2012; Costantini et al., 2014; Borsboom et al., 2011; Epskamp et al., 2012; van der Maas et al., 2006) and a mathematical characteristic of networks (Golino & Epskamp, 2016). Specifically, Golino & Epskamp (2016) demonstrated the two principles following:

1. If the latent factors are orthogonal to each other, the resulting network model consists of unconnected clusters.

2. Assuming factor loadings and residual variances are reasonably on the same scale for every item, the off-diagonal blocks of the variance–covariance matrix will be scaled closer to zero than the diagonal blocks of the variance–covariance matrix. Hence, the resulting network model will contain weighted clusters for each factor.

There are several ways to estimate network models. In one of them, partial correlation coefficients can be used to build networks in which each edge represents the association between two variables conditioned on all other variables (Epskamp & Fried, 2016). Epskamp and Fried (2016) argue that one of the main issues in using partial correlation coefficients to estimate network models is that even when two variables are conditionally independent, the estimated partial correlation coefficient is not zero due to sampling variation. Thus, partial correlation coefficients can reflect spurious correlations, representing relationships that are not true in reality (Epskamp & Fried, 2016). Regularization techniques can be used to deal with spurious connections, such as the least absolute shrinkage and selection operator (LASSO; Tibshirani, 1996), one of the most prominent methods for network estimation on psychological datasets (van Borkulo et al., 2014; Kossakowski et al., 2015; Fried et al., 2015). The LASSO technique avoids overfitting by shrinking the partial correlation coefficients, so small coefficients are estimated to be exactly zero (Golino & Epskamp, 2016; Epskamp & Fried, 2016). This indicates conditional independence and facilitates the interpretability of the network structure, requiring fewer connections to explain the covariance between variables in a dataset (Epskamp & Fried, 2016).

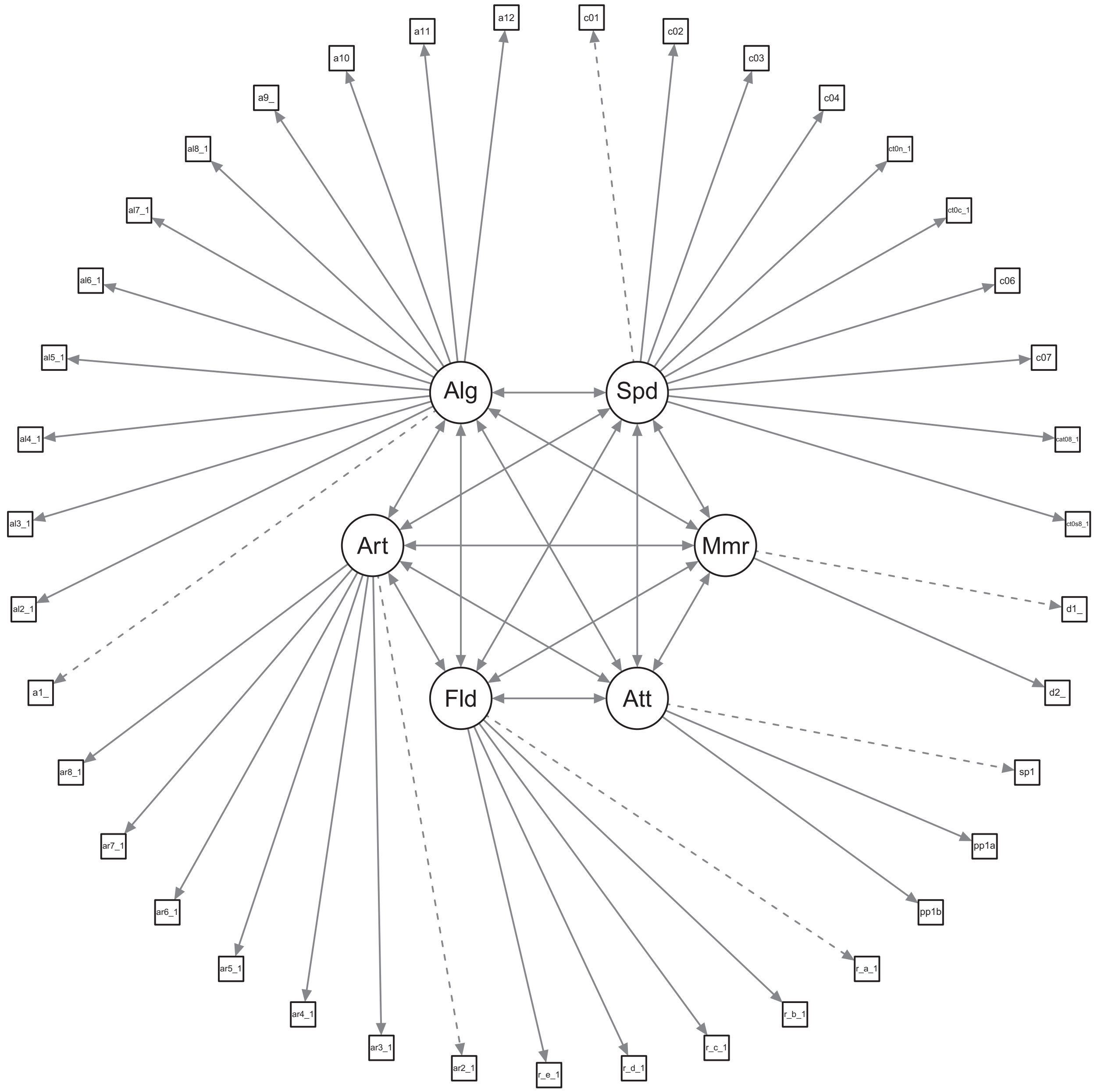

Fig. 3. The theoretical structure of the Žebec et al. (2015) batteries. Spd = speed; Mmr = memory; Att = attention; Fld = fluid reasoning; Art = arithmetic reasoning; Alg = algebraic reasoning.

A new approach to estimate the number of dimensions in psychological data using network psychometrics was proposed by Golino and Epskamp (2016). This is Exploratory Graph Analysis (EGA) and it works as follows. Firstly, it estimates the correlation matrix of the observable variables; then proceeds to use the LASSO estimation to obtain the sparse inverse covariance matrix, with the regularization parameter defined via EBIC over 100 different values. In the last step, the walktrap algorithm (Pons & Latapy, 2005) is used to find the number of dense subgraphs of the partial correlation matrix. In EGA, the number of dense subgraphs identified (i.e., clusters in an undirected weighted network) equals the number of latent factors in a given dataset. It is important to note that EGA is able to both estimate the number of dimensions underlying the data and clearly show which items belong to each dimension.

Golino and Epskamp (2016) studied the accuracy of six techniques to estimate the number of dimensions: (i) very simple structure (VSS; Revelle & Rocklin, 1979) with complexity one; (ii) minimum average partial procedure (MAP; Velicer, 1976); (iii) fit of different number of factors, from 1 to 10, via BIC and via EBIC; (iv) Horn's Parallel Analysis (PA; Horn, 1965) using the generalized weighted least squares factor method; (v) Kaiser-Guttman eigenvalue greater than one rule (Guttman, 1954; Kaiser, 1960), and (vi) EGA. These authors simulated 32,000 data sets with known factor structures, varying the number of factors (2 and 4), the number of dichotomous items (5 and 10), sample size (100, 500, 1000 and 5000) and correlation between factors (orthogonal, .20, .50 and .70), resulting in 64 different conditions. For each condition, 500 data sets were simulated. Golino and Epskamp (2016) showed that EGA (Mean Accuracy = .96, SD = .19) performed comparably to PA (Mean Accuracy = .97, SD = .16) and EBIC (Mean Accuracy = .97, SD = .16) in the two-factor structure, irrespective of the correlation between factors, sample size or number of items. However, EGA outperformed VSS (Mean Accuracy = .22, SD = .41), MAP (Mean

Table 1
Percent accuracy of extraction methods in recovering the correct number of factors under various conditions, $N = 500$, in the original study of Keith et al. (2016): rows 1 to 7. Percent accuracy of exploratory graph analysis (row 8).

Method	Indicators per factor						Mean	Standard
	2	3	4	6	8	10	accuracy	deviation
PA-PCA mean	0	0	0	0	25	100	20.83	36.56
MAP	0	0	0	0	0	0	0.00	0.00
Eigenvalue	0	5	60	0	0	0	10.83	22.06
ML	0	0	0	20	20	20	10.00	10.00
PA-PAF	5	10	15	85	80	100	49.17	39.73
CFA change chi	0	35	70	100	100	100	67.50	38.27
CFA AIC	5	50	90	100	100	100	74.17	35.64
EGA	100	100	100	100	100	100	100.00	0.00

Note: Methods used to determine the number of factors (factor extraction) were PA-PCA: parallel analysis based on principal components analysis; MAP: minimum average partial criterion; eigenvalue: Kaiser–Guttman eigenvalue greater than one criterion; ML: exploratory maximum-likelihood; PA-PAF: parallel analysis based on principal axis factor analysis; CFA change chi: confirmatory factor analysis with the number of factors based on the statistical significance of change in chi-squared; CFA AIC: confirmatory factor analysis using the Akaike information criterion; EGA: exploratory graph analysis.

Accuracy = .78, SD = .41), Kaiser-Guttman eigenvalue rule (Mean Accuracy = .86, SD = .35) and BIC (Mean Accuracy = .92, SD = .27) when the number of factors was two. However, differences between PA and EGA emerged in the four-factor structure. In general, EGA reached a mean accuracy of 89% (SD = 31%) and PA reached a mean accuracy of 80% (SD = 40%). When the correlation between factors was high (.70) and the number of items per factor was 5, only EGA was able to correctly estimate the number of dimensions, achieving a mean accuracy of 53% with a sample size of 1000 and 100% with a sample size of 5000. Golino and Epskamp (2016) also verified how the controlled conditions (sample size, correlation between factors and number of items) and their high-order interactions affected the mean accuracy of each method, via ANOVA. The results showed that EGA was the only technique attaining a high partial eta squared effect size in only one condition (sample size). The other techniques attained high effect sizes from three to nine conditions or their high-order interactions.

2. Methods

We adopted two strategies to show how EGA may be used to explore the underlying dimensionality of intelligence data. The first used the datasets simulated by Keith, Caemmerer, and Reynolds (2016) where PA and MAP attained low accuracies in estimating the correct number of dimensions. They were as follows: (i) sample size of 500 cases, (ii) high correlation between factors (.70), (iii) low factor loading (.50), and (iv) different numbers of indicators per factor (2, 3, 4, 6, 8 or 10). In sake of the present aims, the number of dimensions for each simulated dataset was estimated using EGA, and its accuracy was compared to the accuracy found in the original paper. It is important to note that in the original paper all data were simulated using the Monte Carlo command in the Mplus program, but no further information regarding the simulation was provided (Keith et al., 2016).

The second strategy used EGA to explore the dimensionality of three previously published empirical datasets (Must & Must, 2013, 2014; Demetriou & Kazi, 2006; Žebec et al., 2015) and CFA to test the EGA findings.

We applied two rules to explore the dimensionality of the empirical datasets using EGA (see details and technical instructions in Appendix A):

1) EGA was applied recursively, so that dimensions were analyzed item by item. If a dimension was represented by only one item, this item was deleted from the analysis and EGA was re-run. This process was applied repeatedly until all dimensions were represented by at least two items;
2) If only two items from dimension *A* were identified as part of a different dimension, *B*, these items were deleted and EGA was re-run.

The final solution generated by EGA was submitted to a confirmatory factor analysis using the *robust weighted mean square* estimator (WLSMV) via *lavaan* (Rosseel, 2012). The fit of the models was verified using the root mean-square error of approximation (RMSEA), the comparative fit index (CFI: Bentler, 1990), the normed fit index (NFI), the nonnormed fit index (NNFI: Bentler, 1990) and the Standardized Root Mean Square Residual (SRMR). The model fit is considered good if

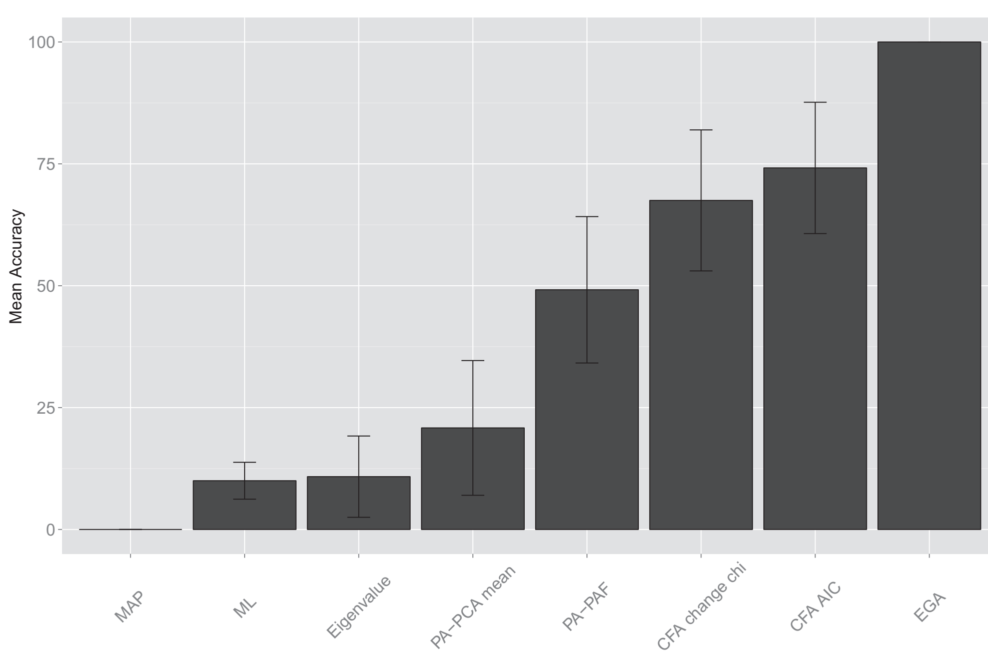

Fig. 4. Mean accuracy and error across methods in estimating the correct number of factors in the simulated datasets of Keith et al. (2016). PA-PCA: parallel analysis based on principal components analysis; MAP: minimum average partial criterion; eigenvalue: Kaiser–Guttman eigenvalue greater than one criterion; ML: exploratory maximum-likelihood; PA-PAF: parallel analysis based on principal axis factor analysis; CFA change chi: confirmatory factor analysis with the number of factors based on the statistical significance of change in chi-squared; CFA AIC: confirmatory factor analysis using the Akaike information criterion; EGA: exploratory graph analysis.

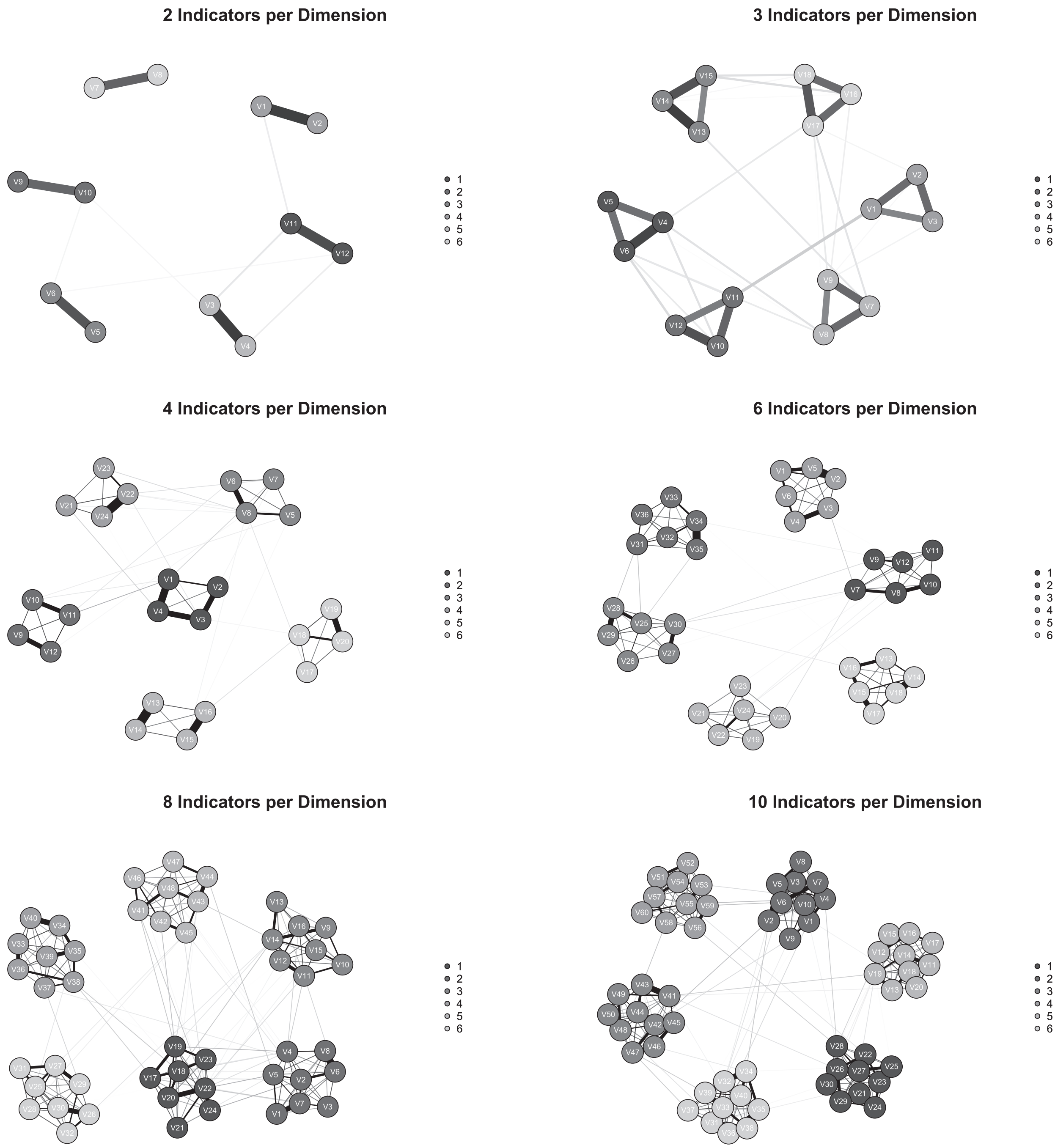

Fig. 5. EGA estimated dimensions for one simulated dataset (out of 20) by each number of indicator per factor.

RMSEA < .06 (Browne & Cudeck, 1993), CFI > .95 (Hu & Bentler, 1999), NFI and NNFI > .90 (Bentler & Bonett, 1980) and SRMR < .08 (Hu & Bentler, 1999).

Finally, we compared the fit of the structure generated by EGA to the structure hypothesized by theory using confirmatory factor analysis via the *lavaan* (Rosseel, 2012) package. All figures showing the standardized weights of CFA models were created using *semPlot* (Epskamp, 2014). EGA was applied using the *EGA* package (Golino, 2016).

The description of the instruments used in each paper is in the following section. The theoretical model underlying each test/battery is graphically presented (i.e. the CFA structure) after the description of the variables used in each study.

2.1. Description of the empirical datasets

2.1.1. Dataset by Must and Must

The Estonian version (Must & Must, 2013, 2014) of the National Intelligence Test (NIT; Haggerty, Terman, Thorndike, Whipple, & Yerkes, 1920) measures intelligence of schoolchildren (grades 3 to 8). This battery is composed by five paper-and-pencil timed subtests, with two complementary scales (A and B). The dataset used in the current paper involved 1802 Estonian children and teenagers who answered form A in 1936 and 2006. In the current paper, we directly analyze the NIT items, not subtest scores. This dataset was firstly used by Must and Must (2013) and subsequently published by Must and Must

Table 2
Distribution of NIT subtests' items by dimension in each step of the exploratory graph analysis. At Step 1 two items were removed (first rule) and at Step 2 two more items were removed (second rule). Eliminated items are colored.

EGA Step 1		EGA Step 2		EGA final structure	
Items	Dimension	Items	Dimension	Items	Dimension
ar3	1	ar15	1	ar1	1
ar4	1	ar16	1	ar2	1
sc11	2	c1	1	ar3	1
sc17	2	c2	1	ar4	1
sc18	2	c3	1	ar5	1
sc20	2	c4	1	ar6	1
c14	2	c5	1	ar7	1
c16	2	c6	1	ar8	1
c17	2	c7	1	ar9	1
c18	2	c8	1	ar10	1
c19	2	c9	1	ar11	1
c20	2	c10	1	ar12	1
c21	2	c12	1	ar13	1
c22	2	c13	1	ar14	1
c23	2	c15	1	sc17	2
c24	2	ar1	2	sc18	2
ar1	3	ar2	2	sc19	2
ar2	3	ar3	2	sc20	2
ar5	3	ar4	2	c14	2
ar6	3	ar5	2	c16	2
ar7	3	ar6	2	c17	2
ar8	3	ar7	2	c18	2
ar9	3	ar8	2	c19	2
ar10	3	ar9	2	c20	2
ar11	3	ar10	2	c21	2
ar12	3	ar11	2	c22	2
ar13	3	ar12	2	c23	2
ar14	3	ar13	2	c24	2
sc4	3	ar14	2	c1	3
sc7	3	sc2	2	c2	3
sc9	3	sc3	2	c3	3
sc13	3	sc4	2	c4	3
c1	4	sc6	2	c5	3
c2	4	sc7	2	c7	3
c3	4	sc8	2	c8	3
c5	4	sc9	2	c9	3
c7	4	sc10	2	sc2	4
c8	4	sc11	2	sc3	4
c9	4	sc12	2	sc4	4
sc2	5	sc13	2	sc6	4
sc3	5	sc14	2	sc7	4
sc6	5	sc15	2	sc8	4
sc8	5	sc16	2	sc9	4
sc10	5	sc17	3	sc10	4
sc12	5	sc18	3	sc11	4
sc14	5	sc19	3	sc12	4
sc15	5	sc20	3	sc13	4
sc16	5	c11	3	sc14	4
sc19	5	c14	3	sc15	4
c11	5	c16	3	sc16	4
ar15	6	c17	3	c6	5
ar16	6	c18	3	c10	5
c4	6	c19	3	c11	5
c6	6	c20	3	c12	5
c10	6	c21	3	c13	5
c12	6	c22	3	c15	5
c13	6	c23	3		
c15	6	c24	3		
sc1	7				
sc5	8				

Note: ar1–ar16 = items from the arithmetic reasoning subtest; sc1–sc20 = items from the sentence completion subtests; c1–c24 = items from the concepts subtests.

(2014). Subtests are described in Must and Must (2014). Only the first three subtests were used in the current paper:

1) Arithmetical Reasoning (A1). The subtest consists of 16 items requiring to specify an unknown quantity. For example: "How many seats are there in 7 rooms, if each room has 30 seats?"
2) Sentence Completion (A2). The subtest consists of 20 items requiring to fill in missing words to make a sentence understandable and correct. An example: "The letter …… came …… good news".
3) Concepts (A3). The subtest consists of 24 items requiring the selection of two characteristic features among several options. For example, "apple: basket/redness/seeds/skin/sweetness".

The theoretical structure of the NIT subtests used in the current study is shown in Fig. 1. The latent variable of arithmetic reasoning (AR) accounts for the arithmetic reasoning items, the latent variable of sentence completion (SC) accounts for the sentence completion items, and the latent variable of concepts (CON) accounts for the concepts items.

2.1.2. Dataset by Demetriou and Kazi (2006, Study 2)

Demetriou and Kazi (2006) examined a total of 840 participants, about equally drawn among 10–15-year-old participants. SES and gender were about equally represented in all but the last age groups. An outline of the cognitive tasks and scoring procedures used is given below and the reader is referred to the original article for details.

2.1.2.1. The battery

2.1.2.1.1. Quantitative reasoning tasks. The quantitative tasks addressed two types of mathematical thought: Proportional reasoning and algebraic reasoning.

2.1.2.1.2. Proportional reasoning tasks. There were two tasks varying systematically in difficulty. The first involved two (i.e., 2/4 to 3/6) and the second involved three factors (i.e., 2/4 to 3/6 to 1/2) varying along their dimension of measurement. Scoring varied from 0 to 2, reflecting the grasp of the proportional relations involved.

2.1.2.1.3. Algebraic reasoning tasks. Participants solved three equations varying in difficulty (i.e., (i) specify x, given that $x = y + 3$ and $y = 1$; (ii) specify x, given that $x = y + u$ and $x + y + u = 30$; (iii) when is it true that $L + M + N = L + P + N$?). Responses were scored as 0 (wrong) or 1 (correct responses).

2.1.2.1.4. The causal-scientific reasoning tasks. Three tasks addressed causal-scientific reasoning: Isolation of variables, hypothesis testing, and integration of hypothesis with data into a model. Scoring (0–2) reflected understanding that to test the effect of a factor one needs to vary this factor while keeping all other factors constant; to test a hypothesis one needs to systematically manipulate the factors involved; translating a hypothesis to a model needs to verify the expected relation in a properly controlled experiment.

2.1.2.1.5. The spatial-imaginal tasks. Two types of spatial-imaginal tasks were used: Mental rotation and visual memory tasks. The mental rotation tasks asked participants to identify or visualize two- or three-dimensional objects under various orientations or rotations. To test imaginal memory, a visual memory task was selected from the Kit of Factor Referenced Tests (Ekstrom, French, & Harman, 1976), where participants identified target figures among a larger set of figures. Scoring (0–2) reflected accuracy and resolution of visual images the participant can produce.

2.1.2.1.6. Social thought tasks. Two types of tasks addressed social thought: Interpersonal relationships and relativistic thinking:

The *interpersonal relationships tasks* required to grasp the relations between intentions, behaviors, and effects of behaviors on a person. Scoring (0–2) reflected the ability to go beyond observable behavior to underlying factors, such as intentions, motives, and moral principles.

Relativistic thinking tasks required to understand the relations between personal behavior and its social consequences under various social frameworks. Participants were asked to specify alternative sides of an argument, explain differences between arguments, and associate different social interests with different arguments. Scoring (0–2) reflected understanding of the various dimensions involved in an issue, integration into a cohesive argument, and contrast arguments according to alternative criteria.

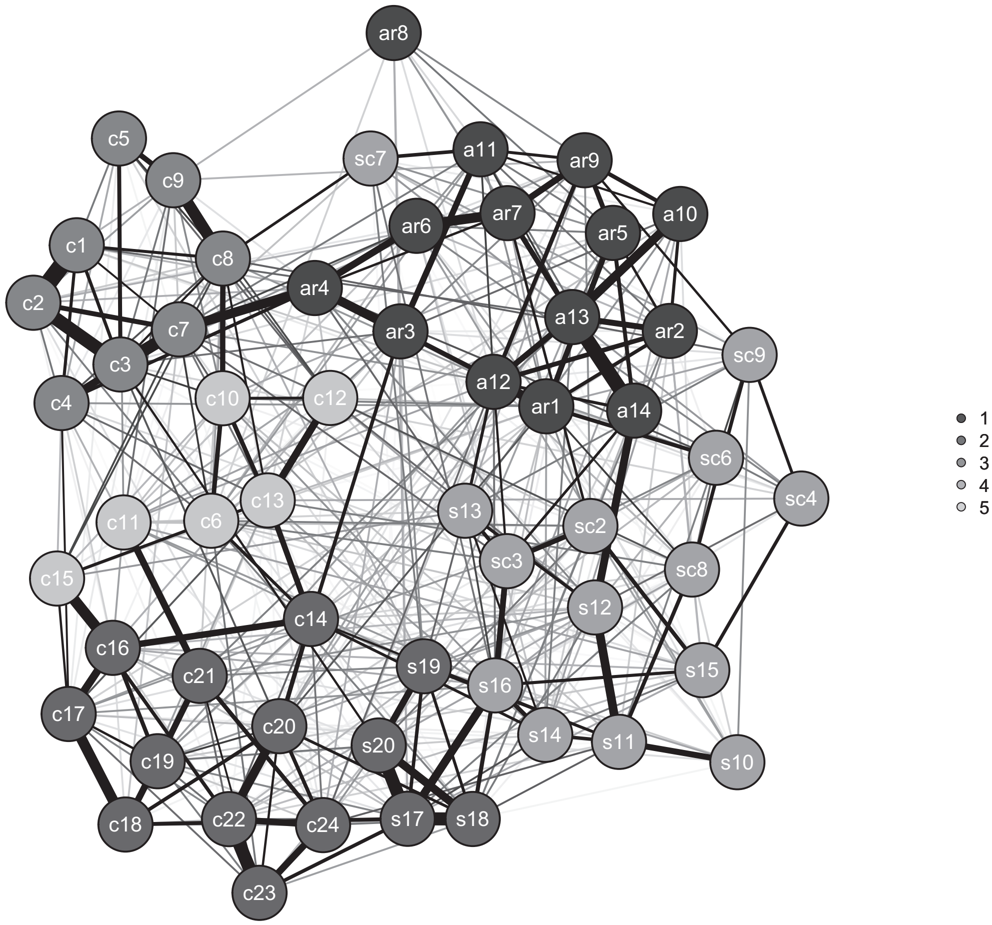

Fig. 6. Network of partial correlations, estimated using graphical lasso, showing the pattern of NIT subtest items per cluster. Each cluster represents a dimension: cluster 1 = arithmetic reasoning; cluster 2 = concepts n.1; cluster 3 = concepts n.2; cluster 4 = sentence completion; cluster 5 = concepts n.3.

2.1.2.1.7. Drawing. Participants were asked to produce two drawings differing in complexity of the objects involved and perspective (i.e., man and a woman are standing hand-in-hand and three or more boats on the water at sunset, some close and others further away) Scoring (0–4) reflected the ability to produce all objects intended in the proper relations to each other in the three-dimensional space.

2.1.2.1.8. Creativity and ideational fluency. To test creativity, the Ornamentation and the Symbol test were selected from the Kit of Factor Referenced Tests (Ekstrom et al., 1976). These tests tap ideational fluency, which is regarded as a component of creativity. The ornamentation test addressed figural fluency (participants decorated each of 24 variants of the same object with a different decorating figure). The symbol test addressed conceptual fluency (participants drew up to five symbols for each of various concepts, such as library, sad, rush, post office, happy, etc.). Performance on these tests was scored according to the criteria described in the Kit of Factor Referenced Tests (Ekstrom et al., 1976). There were two scores for each test varying from 0 to 24 (ornamentation test) or 0–25 (symbol test).

Participants were tested in groups during school hours. The presentation order of tasks was counterbalanced across participants. Cronbach's alpha was .79. The theoretical structure of the instruments used by Demetriou and Kazi (2006) and tested by CFA in the current study is shown in Fig. 2. Seven latent variables were found: spatial, social, and algebraic reasoning, drawing ability, and two aspects of creativity, figural and conceptual fluency.

2.1.3. Dataset by Žebec et al. (2015)

Žebec et al. (2015) examined a total of 478 participants (52% male), drawn from each of the age years 7–17 years, twice in a 12-month interval. In the current paper we analyze the data of the first wave only.

2.1.3.1. Simple reaction time (cat tasks). Participants were asked to recognize the ink color of word-like sets of Xs, recognize color words, choose between responses according to the stimuli presented, and respond to congruent and incongruent Stroop-like stimuli (Cronbach's alpha = .95).

2.1.3.2. Divided attention (pp tasks). This test required simultaneous responding to two different tasks on the two panels, where the stimuli were presented in fast succession (50 to 250 ms). Task 1 was a simple reaction time task as above. Task 2 was an object size classification task. Participants were asked to respond to Task 1 with the left hand on Panel 1, and on to Task 2 with the right hand on Panel 2. (Cronbach's alpha = .93).

2.1.3.3. Working memory (ds tasks). The extended version of forward (FDS) and backward digit span (BDS) test included in the Wechsler intelligence scale test for children (WISC) was used to measure components of working memory. The FDS test comprised 10 pairs of digit sequences varying from 2 to 11 digits. The BDS test included 8 pairs

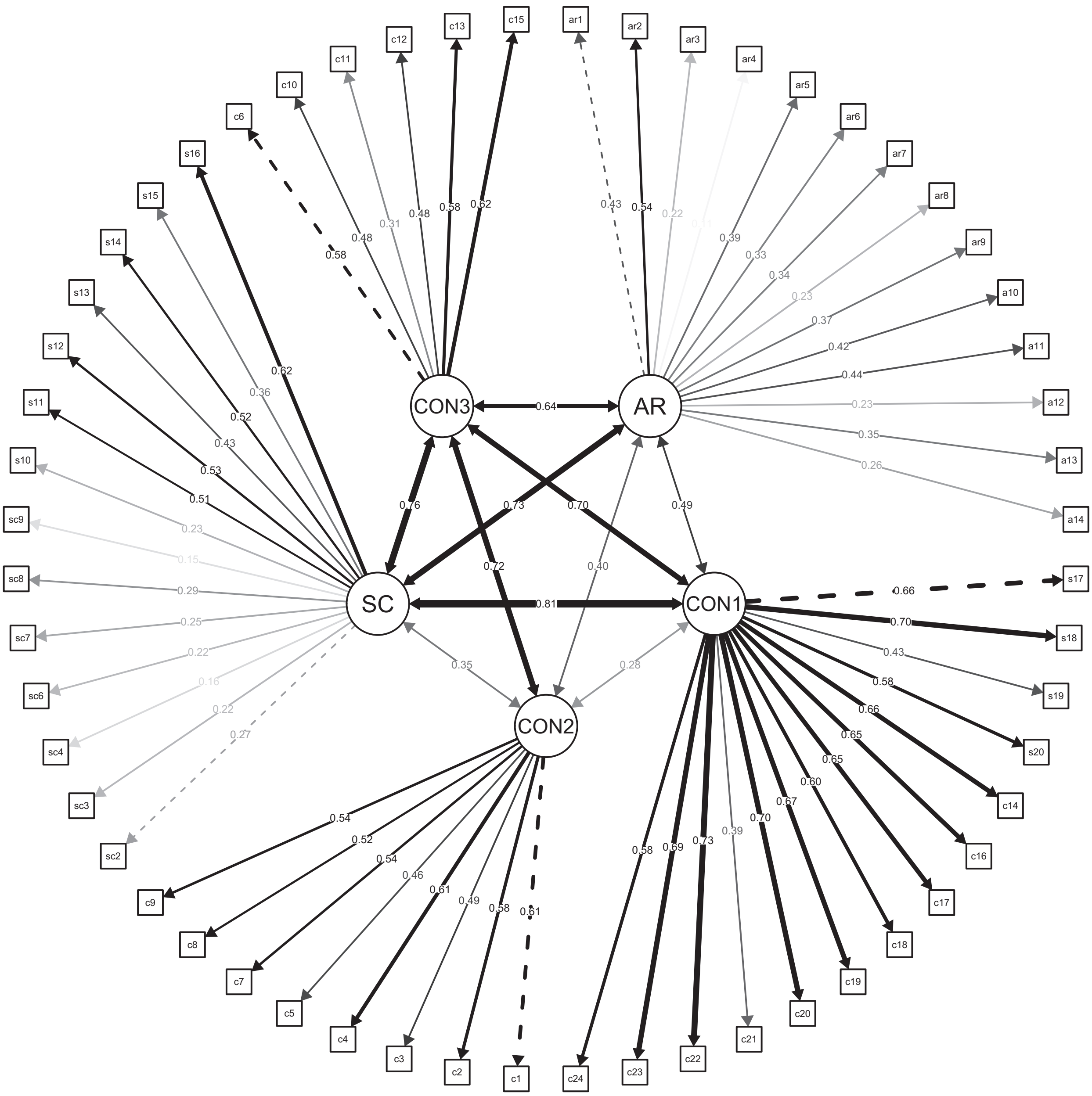

Fig. 7. Standardized weights of the CFA model from the structure suggested by EGA in the NIT subtests' data. AR = arithmetical reasoning; CON1 = concepts n.1; CON2 = concepts n.2; SC = sentence completion; CON3 = concepts n.3.

of digit sequences, varying from 2 to 9 digits (Cronbach's alpha = .88).

2.1.3.4. Raven's Standard Progressive Matrices (rb tasks). Raven's Standard Progressive Matrices were used to address fluid intelligence. SPM include five sets of matrices of increasing complexity (i.e., grasp relations of figures varying along one dimension, two familiar and obvious dimensions, one or more implied dimensions, and several systematically transformed dimensions) (Cronbach's alpha = .94).

2.1.3.5. Mathematics. A paper-and-pencil battery of mathematical reasoning was used. The battery addressed arithmetic and algebra. Items in each domain were scaled along four levels of difficulty. In the arithmetic tasks (arit tasks), participants were asked to specify the operations missing from simple arithmetic equations: One (e.g., 5 ∗ 3 = 8), two (e.g., {4 # 2} ∗ 2 = 6), three (e.g., {3 ∗ 2 # 4} @ 5 = 7), and four operations (e.g., {5 @ 2} o 4 = {12 $ 1} ∗ 2) were missing from the items of each level.

The algebraic reasoning tasks (alg tasks) required to specify one or more unknowns in an equation (e.g., a + 5 = 8, a = ? ; u = f + 3; f = 1; u = ? ; if (r = s + t) and (r + s + *t* = 30), specify r = ?; when is true that {L + M + N} = {L + P + N}?) (Cronbach's alpha of the whole mathematical reasoning test = .93).

The theoretical structure of these batteries was tested by CFA and it is illustrated in Fig. 3. Six latent variables were found: speed, memory, attention, fluid reasoning (Raven), arithmetic, and algebraic reasoning.

3. Results: simulated datasets

3.1. Datasets simulated by Keith, Caemmerer, and Reynolds (2016)

Keith et al. (2016) estimated the number of factors in simulated datasets with different number of cases, different correlations between factors, and different factor loadings and number of indicators per factor. Their analysis pointed to a low accuracy of the methods they applied

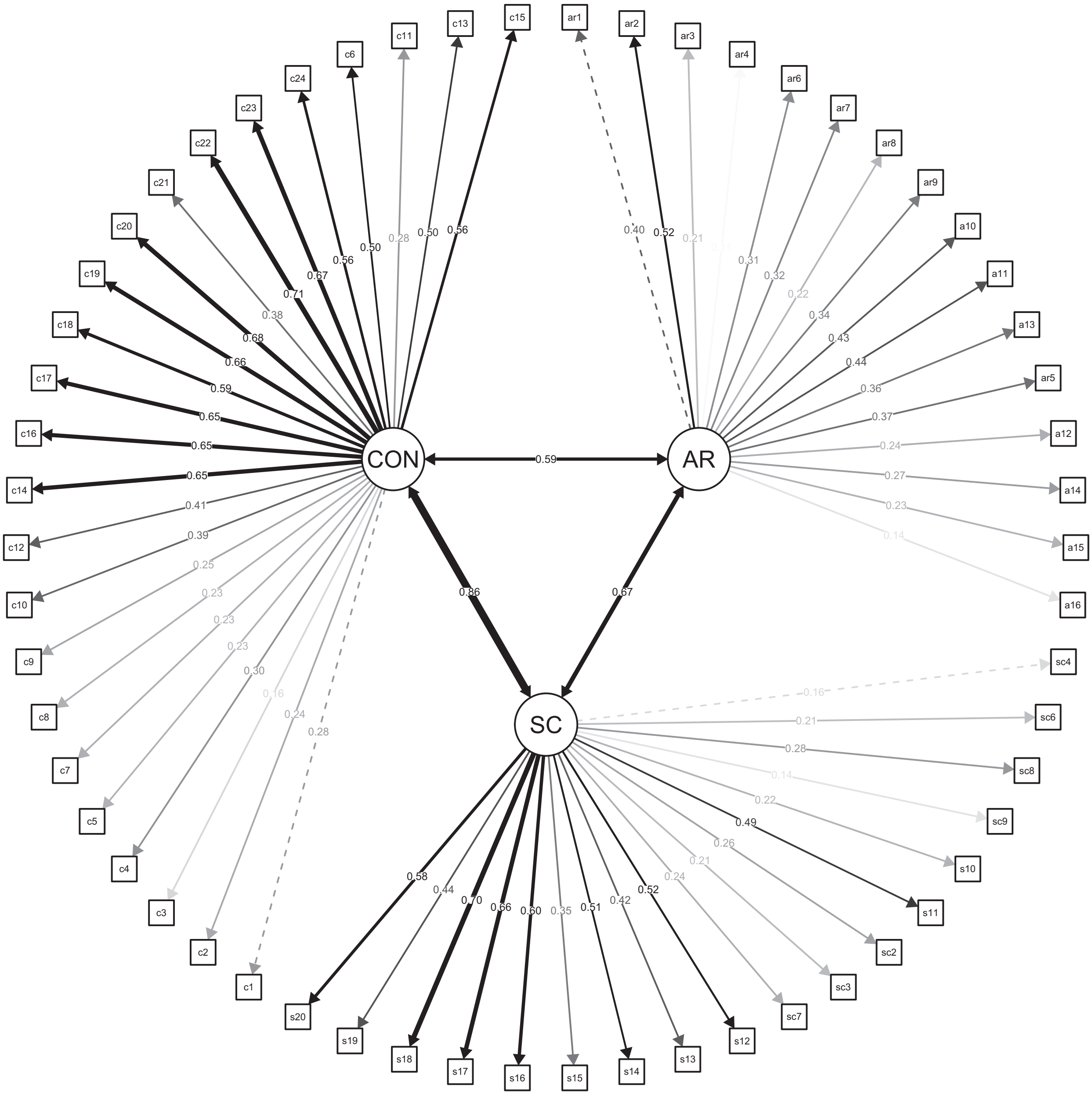

Fig. 8. Standardized weights of the NIT subtests' confirmatory factor model from the structure expected by theory. AR = arithmetical reasoning; CON = concepts; SC = sentence completion.

to recover the number of factors when the simulated data presented (i) 500 cases, (ii) high correlation between factors (.70), (iii) low factor loading (.50), and (iv) different number of indicators per factor (i.e., 2, 3, 4, 6, 8 or 10). They found a factor recovery mean accuracy of zero for MAP, irrespective of the number of indicators per factor (true number of factors = 6). Maximum likelihood exploratory factor analysis attained a mean accuracy of 10% (SD = 10%); when the Kaiser–Guttman criterion for retaining factors with an eigenvalue >1 was applied, a mean accuracy of 10.83% (SD = 22.06%) was attained. Parallel analysis by PCA attained a slightly better accuracy, varying from zero (when they were from 2 to 6 indicators per factor) to 100% (10 indicators per factor), with a mean accuracy of 20.83% (SD = 36.56%); parallel analysis by PAF attained a mean accuracy of 49.17% (SD = 39.73%). Two approaches using confirmatory factor analysis (i.e., specifying the number of factors (i) based on the statistical significance of chi-square change and (ii) using the Akaike information criterion) proved to be the best for the determination of the number of factors when between-factor correlations were high (.70), factor loadings were low (.50), and N = 500. These methods attained a mean accuracy of 67.50% (SD = 38.27%) and 74.17% (SD = 35.64%), respectively.

When EGA was applied to the same datasets (20 simulated datasets per condition, in a total of 120 datasets), the estimated number of factors were 6 regardless of the number of indicators per factor (2, 3, 4, 6, 8 or 10). In other words, EGA was able to correctly estimate the number of factors in 100% of the cases. Table 1 shows the accuracy by method used in the original (first seven rows) and the present study (last row). Fig. 4 shows the mean accuracy (and error) by method and Fig. 5 shows the EGA estimated dimensions for one simulated dataset (out of 20) by each number of indicator per factor.

It is recognized that this data set does have several limitations. For instance, the number of replications per condition is rather limited, increasing the standard error of measurements involved. However, we opted to use this dataset as a link between the present paper and the literature on the topic. Specifically, using this data demonstrates, on the one hand, the formal conditions rendering EGA preferable over other methods. On the other hand, our paper also

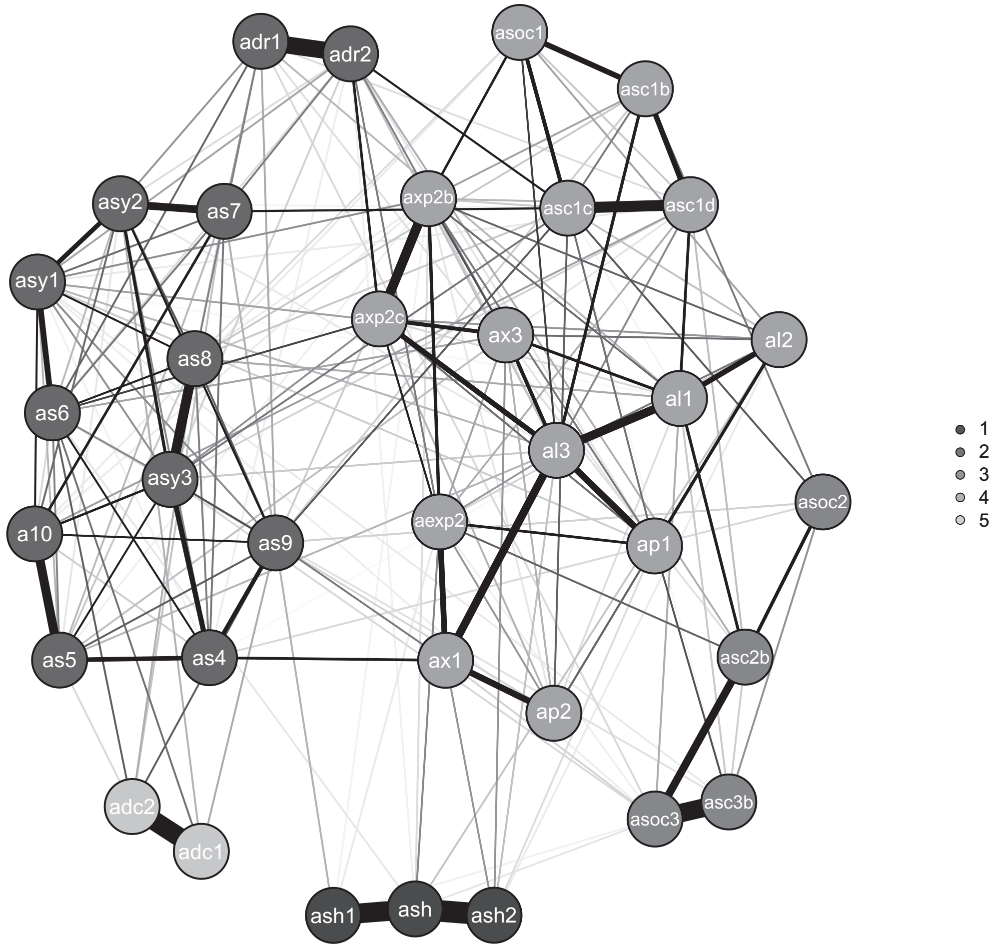

Fig. 9. Dimensions found in the dataset from Demetriou and Kazi's (2006) study using EGA. Each cluster represents a dimension: cluster 1 = spatial ability; cluster 2 = creativity; cluster 3 = social ability; cluster 4 = fluid reasoning; cluster 5 = figural fluency.

demonstrates that EGA is preferable over other methods when applied on real data sets as well.

3.2. Results: real datasets

3.2.1. Dataset by Must and Must (2013, 2014)

The EGA was applied in two steps. At the first step, the number of factors was estimated based on all 60 items included in the three NIT tests used. At first, EGA abstracted nine factors, but three of them were associated with only a single item (Table 2). Thus, at a second step, these three items were dropped from the analysis and EGA was re-ran on the remaining 57 items. In this second analysis, EGA abstracted three dimensions, each with at least 15 items. However, two items from the arithmetical reasoning subtest (ar15 and ar16) were found to be part of a dimension completely formed by items belonging to the concepts subtest. It is reminded that if only two items associated with dimension *A* are found to be part of a different dimension, these items are deleted from the analysis and the EGA is re-run. This analysis abstracted five dimensions (Table 2): (i) Arithmetic reasoning (composed by 14 items from the arithmetic reasoning subtest); (ii) concepts n.1 (composed by four items from the sentence completion subtest and ten items from the concepts subtest); (iii) concepts n.2 (composed by eight items from the concepts subtest); (iv) sentence completion (composed by 14 items from the sentence completion subtest); and (v) concepts n.3 (composed by six items from the concepts subtest). Table 2 shows the distribution of items by dimension in each step of the exploratory graph analysis. At Step 1 two items were removed, following the first rule described in method; at Step 2 two more items were removed following the second rule described in method section. The items eliminated from the analysis, in each step, are colored. The structure suggested by EGA is presented in Fig. 6, which shows five clusters in the estimated partial correlations network, using graphical lasso.

Confirmatory factor analysis showed that the structure of the NIT items suggested by EGA (Fig. 7) has an adequate fit to the data [χ^2 (1474) = 3535.98; p = 0.00; CFI = 0.97; RMSEA = 0.028; NFI = 0.96; NNFI = 0.97, SRMR = .04]. Also, the fit of the structure suggested by theory, which prescribes only three factors (one for each subtest; Fig. 8) was marginally adequate [χ^2 (1592) = 6659.97; p = 0.00; CFI = 0.94; RMSEA = 0.04; NFI = 0.92; NNFI = 0.94, SRMR = .05].

3.2.2. Dataset by Demetriou and Kazi (2006, Study 2)

In the Demetriou and Kazi's (2006) dataset EGA was applied in two steps. In the first steps, all variables were used. However, variables *arot1* and *arot2*, indicators of mental rotation, formed one cluster each. Thus, at the second step of the analysis both variables were removed and EGA was re-ran. In this second analysis, no further exclusion of items was required. This final EGA analysis suggested five dimensions: 1) Spatial ability (associated with three spatial reasoning items); 2) Creativity (associated with two drawing items and ten symbol items); 3) Social reasoning (associated with four items of the social ability test); 4) Fluid reasoning (associated with three algebra items, two isolation of variables items, five items from the hypothesis testing instrument and four items of social ability); 5) Figural fluency (associated with two items from the decoration task). Fig. 9 shows the dimensions found using EGA.

Confirmatory factor analysis showed that the structure of Demetriou and Kazi's (2006) data suggested by EGA (Fig. 10) attained an adequate

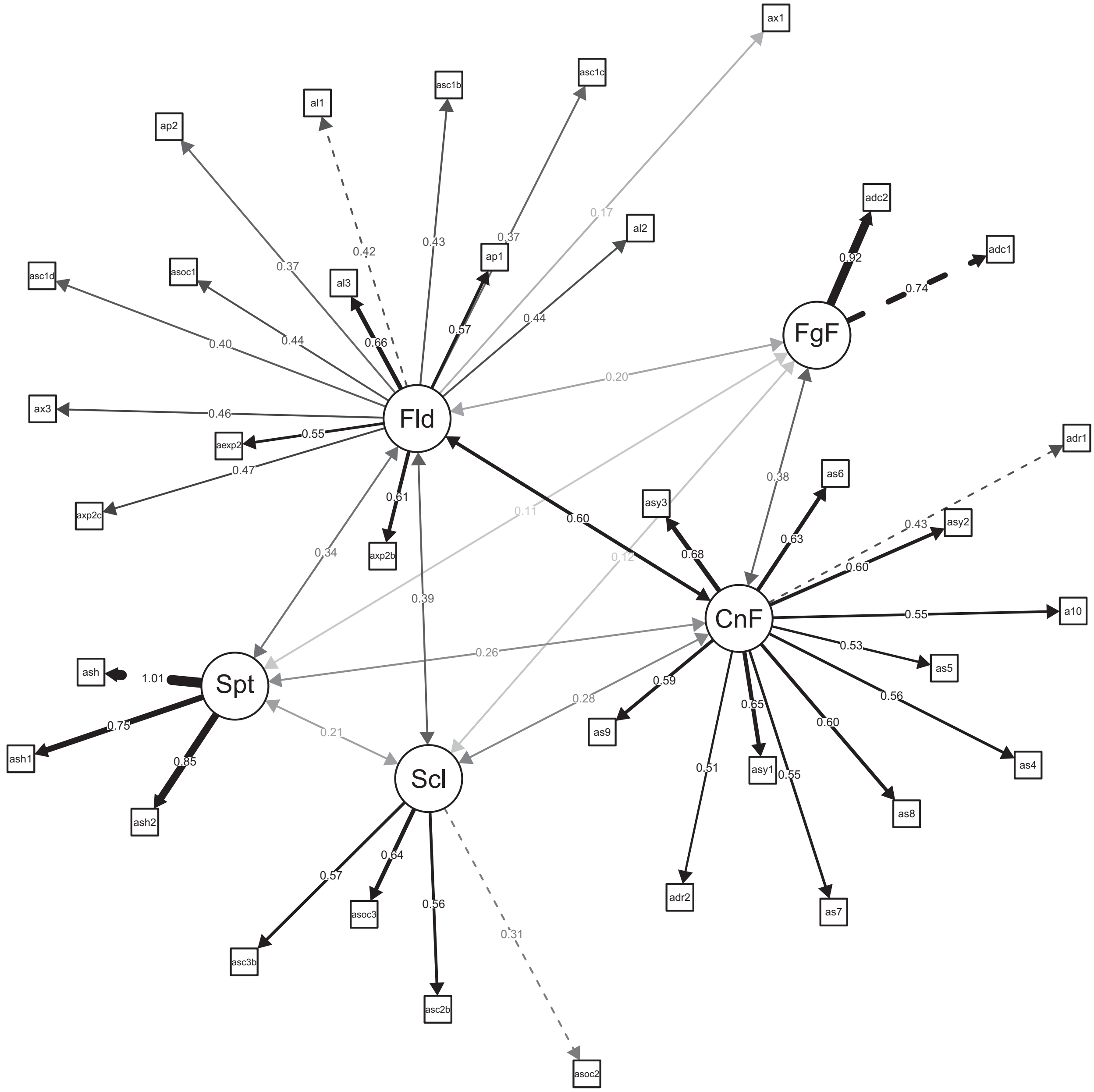

Fig. 10. Standardized weights of the confirmatory factor model from the structure suggested by EGA in the Demetriou and Kazi's (2006) dataset. FgF = figural fluency; CnF = conceptual fluency; Scl = social sbility; Spt = spatial ability; Fld = fluid reasoning.

fit to the data [χ^2 (550) = 769.36; $p = 0.00$; CFI = .99; RMSEA = .02; NFI = .95; NNFI = .98; SRMR = .04]. However, the structure expected by theory (Fig. 11) also presented an adequate data fit [χ^2 (608) = 891.31; $p = 0.00$; CFI = 0.98; RMSEA = 0.02; NFI = 0.94; NNFI = 0.98; SRMR = .04]. The EGA model presents a slightly better fit, since the CFI is higher than in the theoretical model.

3.2.3. Dataset by Žebec et al. (2015)

In the Žebec et al.'s (2015) dataset, EGA was applied in two steps. In the first step, EGA abstracted a structure with three dimensions, but one of them was associated with only a single item (*arit1_1*). This item was removed, and EGA was re-ran. The second analysis indicated four dimensions: 1) Attention (associated with the six cognitive control indicators—choice reaction time, Stroop-like tasks, and three indicators of attention); 2) fluid reasoning (associated with four of the five Raven sets A–D, four arithmetic items and two algebra tasks); 3) processing speed (associated with four processing speed indicators); 4) mathematical reasoning ability (associated with three arithmetic items and ten algebra tasks; this dimension also included Set E of Raven Matrices, which is the most complex part of the Raven test and also the two digit-span indicators). Fig. 12 shows the dimensions found using EGA.

Confirmatory factor analysis showed that the structure of Žebec et al.'s (2015) data suggested by EGA (Fig. 13) attained an adequate fit to the data [χ^2 (623) = 734.73; $p = 0.00$; CFI = .998; RMSEA = 0.02; NFI = 0.99; NNFI = 0.99]. However, the structure expected by theory (Fig. 14) also attained an adequate data fit [χ^2 (687) = 905.67; $p = 0.00$; CFI = 0.99; RMSEA = 0.03; NFI = 0.99; NNFI = 0.99].

4. Discussion

This study suggested that EGA is closer than other competing methods to psychology's ideal for a method that would accurately capture the dimension underlying behavior and cognitive ability. We

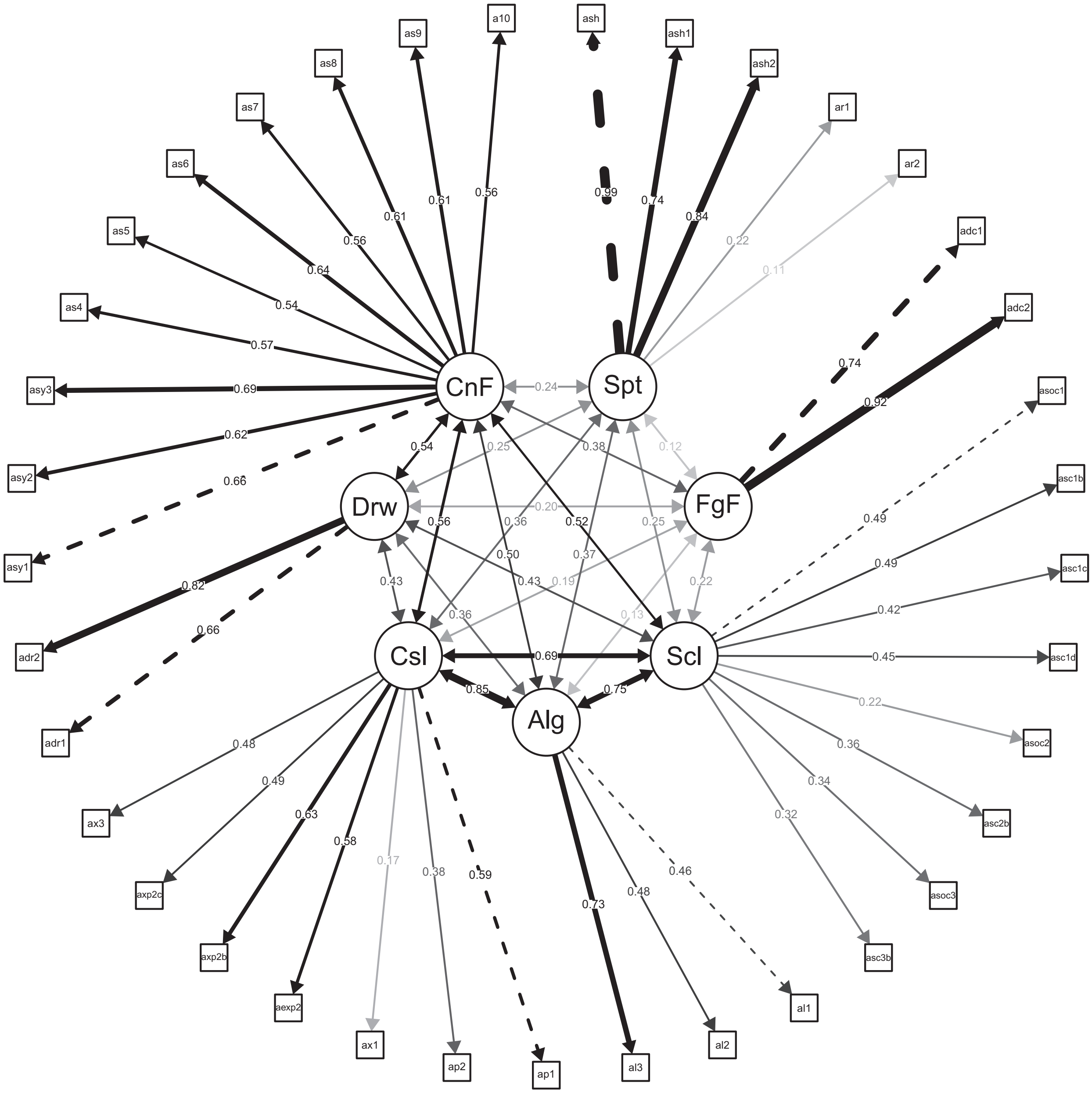

Fig. 11. Standardized weights of the confirmatory factor model from the structure expected by theory in the Demetriou and Kazi's (2006) dataset. Spt = spatial ability; FgF = figural fluency; Scl = social ability; Alg = algebric reasoning; Csl = causal reasoning; Drw = drawing ability; Crt = Creativity; CnF = conceptual fluency.

showed that EGA was more accurate than other methods, including confirmatory factor analysis, to reveal the dimensions underlying performance on various cognitive test batteries under a variety of test and sample conditions. Notably, this method was close to CFA in identifying underlying dimensions but it also worked in conditions where CFA failed (i.e. high factor correlation, low factor loading, sample size of 500 and only two indicators per factor). Therefore, our findings are in line with recent research suggesting that EGA is more efficient than other traditional methods, such as parallel analysis and MAP, in estimating the correct number of dimensions underlying simulated datasets mimicking the ones found in empirical research (Golino & Epskamp, 2016). The advantages of EGA, additionally to its accuracy in estimating the correct number of dimensions are as follows:

1) It does not demand a large sample size to correctly estimate the number of dimensions or factors even when the correlations between them are high (.7), the factor loadings are low (.5), and the number of indicators per factor vary (from 2 to 10);
2) It estimates the number of dimensions/factors using a combination of penalized maximum likelihood estimation (via graphical LASSO) and a random walk algorithm (i.e. walktrap). These properties provide the following advantages:
   a. It decreases overfitting, since the LASSO shrinks low partial correlations to zero;
   b. It facilitates the interpretability of the estimated networks, making it easier to visually identify the cluster of items, which stand for the underlying latent variables;
   c. It suggests structures that are optimized in terms of fit, when assessed by CFA.

Thus, it seems appropriate to use EGA for the identification of factors and SEM/CFA for the specification of between factor relations, especially when the possible direction of between factor relations needs to be specified. This paper presents evidence that EGA offers a robust solution to a serious problem in intelligence research pointed out by Keith et al.

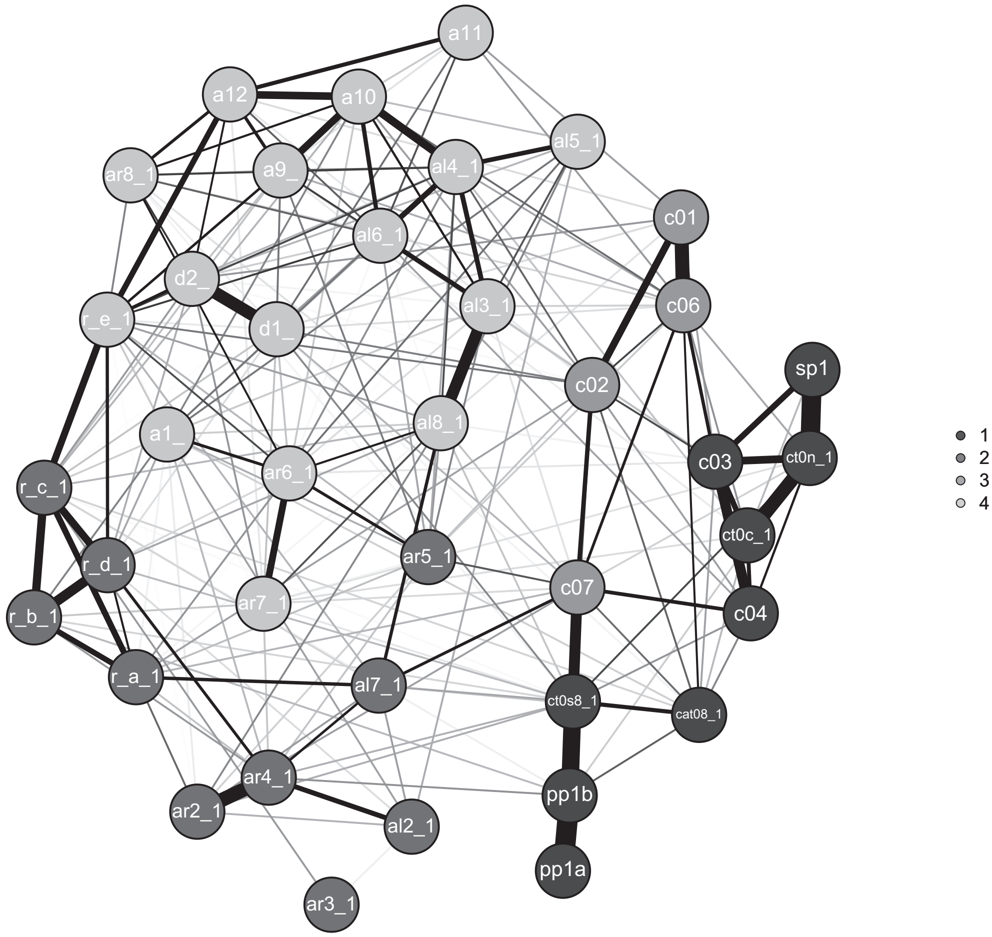

Fig. 12. Dimensions found in the dataset from Žebec et al.'s (2015) using EGA. Each cluster represents a dimension: cluster 1 = attention; cluster 2 = fluid ability; cluster 3 = processing speed; cluster 4 = operational ability.

(2016). Specifically, in face of the limitations of parallel analysis and MAP, these authors suggested that researchers use confirmatory factor analysis (CFA) guided by some formal or informal theory about the data, because CFA was more accurate than the other methods in recovering the correct number of dimensions in their simulation study. We argued in the introduction that theory cannot definitely specify how many dimensions/factors underlie a given instrument or battery and which items really relate to each dimension/factor, even if it is, admittedly, useful when available. We argue, in face of the evidence presented here, that the optimum solution is to use EGA to explore the basic dimensionality of a given instrument and, then, use CFA to verify the fit of the suggested structure.

It is notable that EGA is part of a new area termed network psychometrics (Epskamp et al., 2017), that emerges as a subfield of psychological networks (Epskamp and Fried, 2016), an area of research that uses network modeling for exploratory studies of behavior Borsboom & Cramer, 2013; Schmittmann et al., 2013). Psychological networks suggest new ways for understanding psychological constructs, which may be more informative than traditional latent-variable modeling approach, and may be useful in various fields of research such as psychopathology (Borsboom et al., 2011; Borsboom & Cramer, 2013; Fried et al., 2015; Isvoranu, Borsboom, et al., 2016a; Isvoranu, Borsboom, van Os, & Guloksuz, 2016b), developmental psychology (van der Maas et al., 2006), quality of life (Kossakowski et al., 2015) and cognitive neuroscience (Smith Bassett & Ed Bullmore, 2006). As usual in any new fields, more studies are required to investigate how EGA performs under different conditions, i.e. sample sizes, factor correlations, factor loadings, number of indicators per factors, and so on. Special attention should be given to the number of thresholds (or response categories) by item.

The work of Golino and Epskamp (2016) and the research presented in this paper did not investigate the impact of thresholds in the accuracy of EGA. The focus was on datasets with binary items or items with limited variation which are common in the field of intelligence. However, a large number of instruments used in other fields employ Likert-like items varying along much wider scales. This limitation needs to be removed by future research.

Appendix A. Appendix

The EGA package can be installed in R using the following code:

```
library("devtools")
devtools::install_github('hfgolino/EGA')
```

The EGA package was developed as a simple and easy way to implement the Exploratory Graph Analysis technique. The package has three main functions: EGA, bootEGA and CFA. These functions will be briefly explained below. It is important to note that the *bootEGA* function was not used in the current paper.

EGA: Estimates the number of dimensions of a given dataset/instrument using graphical lasso and a random walk algorithm. The glasso regularization parameter is set via EBIC.

Usage:

```
EGA(data, plot.EGA = TRUE)
```

A.1. Arguments

data: A dataframe with the variables to be used in the analysis.

plot.EGA: Logical. If true, returns a plot of the network of partial correlations estimated via graphical lasso and its estimated dimensions.

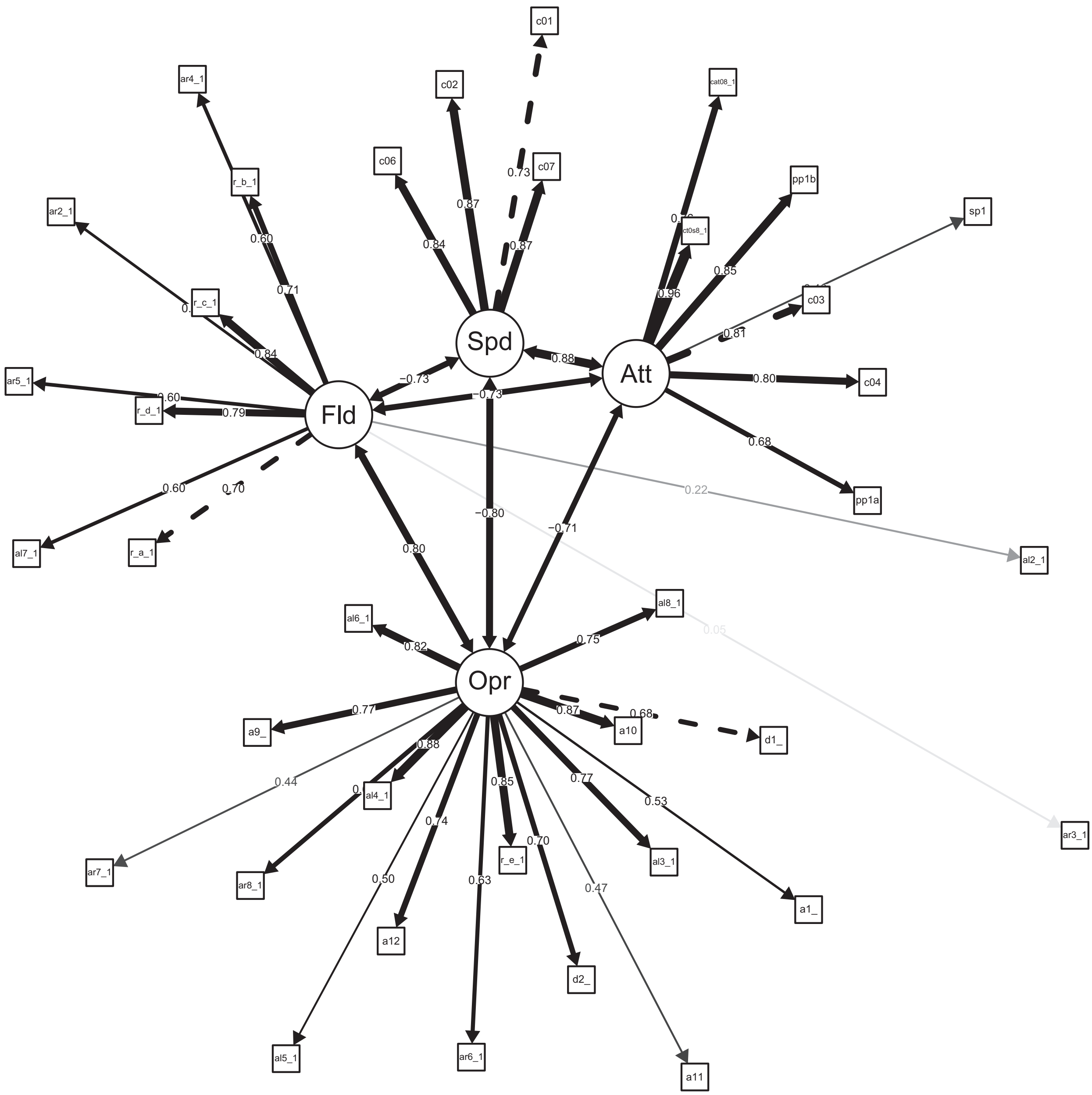

Fig. 13. Standardized weights of the confirmatory factor model from the structure suggested by EGA in the Žebec et al.'s (2015) dataset. Att = Attention; Spd = Processing Speed; Fld = Fluid Ability; Opr = Operational Ability.

Examples with datasets available in the EGA package:

```
ega.wmt <- EGA(data = wmt2[,7:24], plot.EGA = TRUE)
ega.intel <- EGA(data = intelligenceBattery[,7:66], plot.EGA = TRUE)
```

bootEGA: Estimates the number of dimensions of *n* bootstraps from the empirical correlation matrix, and returns a typical network (i.e. the network formed by the median pairwise partial correlations over the n bootstraps) and its dimensionality.

Usage:

```
bootEGA(data, n, medianStructure = TRUE, plot.MedianStructure = TRUE)
```

A.2. Arguments

data: A dataframe with the variables to be used in the analysis.

n: An integer value representing the number of bootstraps.

MedianStructure: Logical. If true, returns the typical network of partial correlations (estimated via graphical lasso), which is the median of all pairwise correlations over the n bootstraps, and estimates its dimensions.

plot.MedianStructure: Logical. If true, returns a plot of the typical network (partial correlations), which is the median of all pairwise correlations over the n bootstraps, and its estimated dimensions.

Examples with datasets available in the EGA package:

```
boot.wmt <- bootEGA(data = wmt2[,7:24], n = 500, medianStructure = TRUE, plot.MedianStructure = TRUE)
boot.intwl <- bootEGA(data = intelligenceBattery[,7:66], n = 500, medianStructure = TRUE, plot.MedianStructure = TRUE)
```

CFA: Verifies the fit of the structure suggested by EGA using confirmatory factor analysis.

Usage:

```
CFA(ega.obj, estimator, plot.CFA = TRUE, data, …)
```

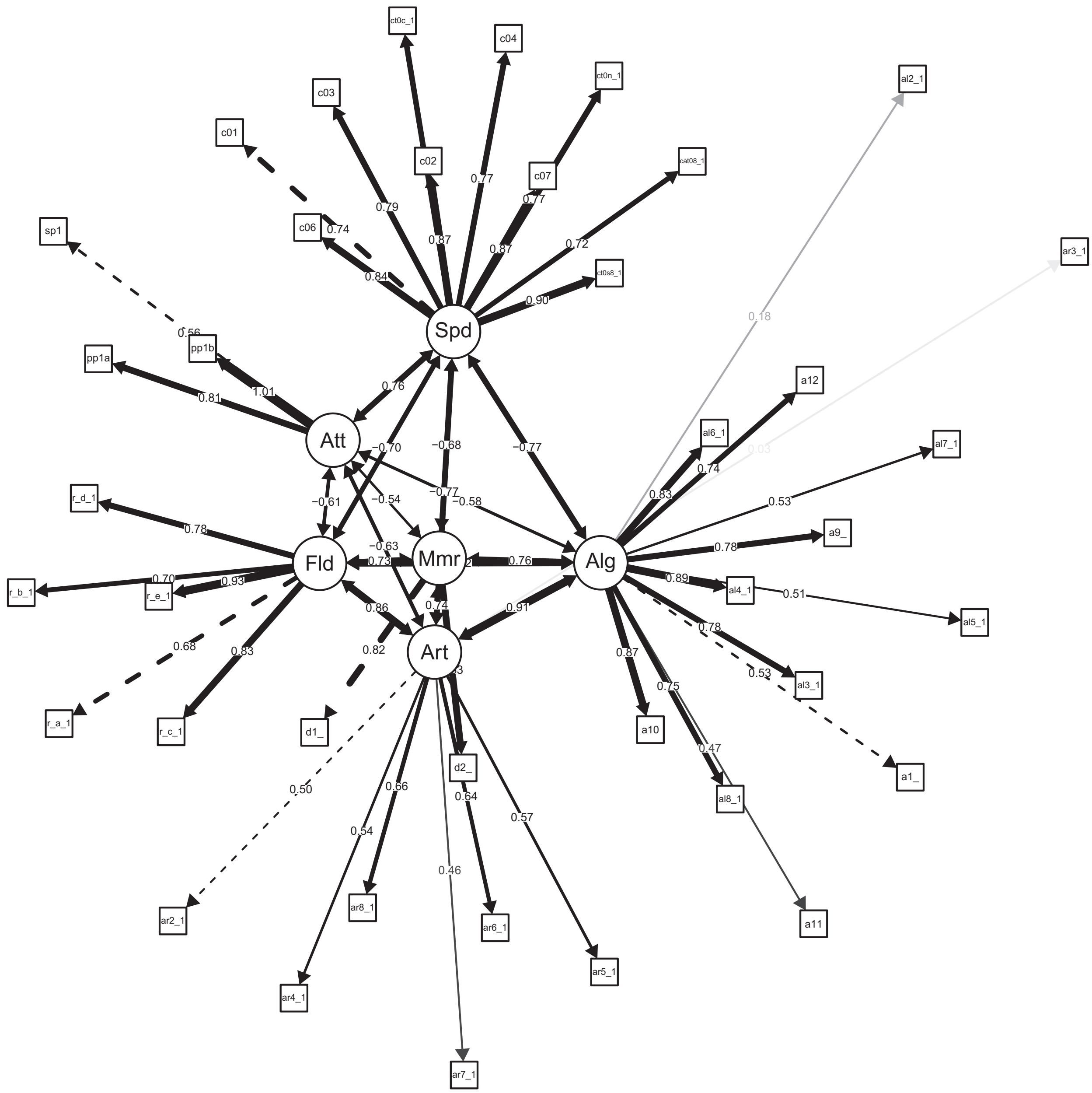

Fig. 14. Standardized weights of the confirmatory factor model from the structure expected by theory in the Žebec et al's (2015) dataset. Att = Attention; Spd = Processing Speed; Fld = Fluid Ability; Mmr = Memory; Alg = Algebra; Art = Arithmetic Ability.

A.3. Arguments

ega.obj: An EGA object.

estimator: The estimator used in the confirmatory factor analysis. "WLSMV" is the estimator of choice for ordinal variables. "ML" or "WLS" for interval variables.

plot.CFA: Logical. Should the CFA structure with its standardized loadings be plot?

data: A dataframe with the variables to be used in the analysis.

…: Other arguments of the *lavaan* package.

Examples with datasets available in the EGA package:

```
ega.wmt <- EGA(data = wmt2[,7:24])
cfa.wmt <- CFA(ega.obj = ega.wmt, estimator = "WLSMV",
plot.CFA = TRUE, data = wmt2)
ega.intel <- EGA(data = intelligenceBattery[,7:66])
cfa.intel <- CFA(ega.obj = ega.intel, estimator =
"WLSMV", plot.CFA = TRUE, data =
intelligenceBattery[,7:66])
```

References

Bentler, P. M. (1990). Comparative fit indexes in structural models. *Psychological Bulletin*, *107*, 238–246.

Bentler, P. M., & Bonett, D. G. (1980). Significance tests and goodness of fit in the analysis of covariance structures. *Psychological Bulletin*, *88*, 588.

Borsboom, D., & Cramer, A. O. J. (2013). Network analysis: An integrative approach to the structure of psychopathology. *Annual Review of Clinical Psychology*, *9*, 91–121.

Borsboom, D., Cramer, A. O. J., Schmittmann, V. D., Epskamp, S., & Waldorp, L. J., (2011). The small world of psychopathology. PLoS ONE, 11, e27407

Browne, M. W., Cudeck, R., Bollen, K. A., & Long, J. S. (1993). Alternative ways of assessing model fit. *Testing structural equation models* (pp. 136–162). Newbury Park: Sage.

Buja, A., & Eyuboglu, N. (1992). Remarks on parallel analysis. *Multivariate Behavioral Research*, *27*, 509–540.

Carroll, J. B. (1993). *Human cognitive abilities: A survey of factor-analytic studies*. New York: Cambridge University Press.

Chandrasekaran, V., Parrilo, P. A., & Willsky, A. S. (2010, September). Graphical model selection via convex optimization. *Communication, control, and computing (Allerton), 2010 48th Annual Allerton conference on* (pp. 1610–1613). IEEE Latent variable.

Costantini, G., Epskamp, S., Borsboom, D., Perugini, M., Mõttus, R., Waldorp, L. J., & Cramer, A. O. (2014). State of the art personality research: A tutorial on network analysis of personality data in R. *Journal of Research in Personality*.

Cramer, A. O. J., van der Sluis, S., Noordhof, A., Wicherts, M., Geschwind, N., Aggen, S. H., ... Borsboom, D. (2012). Dimensions of normal personality as networks in search of equilibrium: You can't like parties if you don't like people. *European Journal of Personality*, *26*, 414–431.

Crawford, A. V., Green, S. B., Levy, R., Lo, W. J., Scott, L., Svetina, D., & Thompson, M. S. (2010). Evaluation of parallel analysis methods for determining the number of factors. *Educational and Psychological Measurement*, *70*, 885–901.

Demetriou, A., & Kazi, S. (2006). Self-awareness in g (with processing efficiency and reasoning). *Intelligence*, *34*, 297–317.

Ekstrom, R. B., French, J. W., & Harman, H. H. (1976). *Manual for kit of factor-referenced tests.* Princeton, NJ: Educational Testing Service.

Epskamp, S. & Fried, E. I. (2016). A primer on estimating regularized psychological networks arXiv preprint Stat-Ap/1607.01367. Available at: http://arxiv.org/abs/1607.013677

Epskamp, S. (2014). semPlot: Path diagrams and visual analysis of various SEM packages' output. *R package version 1.0.1*http://CRAN.R-project.org/package=semPlot

Epskamp, S., Cramer, A. O. J., Waldorp, L.J., Schmittmann, V.D., & Borsboom, D. (2012). qgraph: Network visualizations of relationships in psychometric data. Journal of Statistical Software, 48, 1–18. URL http://www.jstatsoft.org/v48/i04/.

Epskamp, S., Maris, G., Waldorp, L. J., & Borsboom, D. (2017). Network Psychometrics. To appear in In P. Irwing, D. Hughes, & T. Booth (Eds.), *Handbook of psychometrics*. New York: Wiley (in press).

Fried, E. I., Bockting, C., Arjadi, R., Borsboom, D., Tuerlinckx, F., Cramer, A., ... Stroebe, M. (2015). From loss to loneliness: the relationship between bereavement and depressive symptoms. *Journal of Abnormal Psychology*, *124*, 256–265.

Garrido, L. E., Abad, F. J., & Ponsoda, V. (2011). Performance of Velicer's minimum average partial factor retention method with categorical variables. *Educational and Psychological Measurement*, *71*, 551–570.

Golino, H. F. (2016). *EGA package.* Available at github.com/hfgolino/EGA

Golino, H. F., & Epskamp, S. (2016). Exploratory graph analysis: a new approach for estimating the number of dimensions in psychological research. arXiv preprint Stat-Ap/1605.02231. Available at: http://arxiv.org/abs/1605.02231.

Green, S. B., Redell, N., Thompson, M. S., & Levy, R. (2016). Accuracy of revised and traditional parallel analyses for assessing dimensionality with binary data. *Educational And Psychological Measurement*, *76*, 5–21.

Guttman, L. (1954). Some necessary conditions for common-factor analysis. *Psychometrika*, *19*(2), 149–161.

Haggerty, M., Terman, L., Thorndike, E., Whipple, G., & Yerkes, R. (1920). National intelligence tests. *Manual of directionsfor use with scale A, form 1 and scale B, form 1*. New York: World Book Company.

Horn, J. (1965). A rationale and test for the number of factors in factor analysis. *Psychometrika*, *30*, 179–185.

Hu, L. T., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling*, *6*, 1–55.

Isvoranu, A. M., Borsboom, D., van Os, J., & Guloksuz, S. (2016a). *A network approach to environmental impact in psychotic disorder: Brief theoretical Framework.* Schizophrenia Bulletin (Advance Access published May 13, 2016).

Isvoranu, A. M., van Borkulo, C. D., Boyette, L., Wigman, J. T. W., Vinkers, C. H., & Borsboom, D. (2016b). *A network approach to psychosis: Pathways between childhood trauma and psychotic symptoms.* Schizophrenia Bulletin Advance Access published May 10, 2016.

Jensen, A. R. (1998). *The g factor: The science of mental ability*. Westport, CT: Praeger.

Kaiser, H. F. (1960). The application of electronic computers to factor analysis. *Educational and psychological measurement*, *20*, 141–151.

Keith, T. Z., Caemmerer, J. M., & Reynolds, M. R. (2016). Comparison of methods for factor extraction for cognitive test-like data: Which overfactor, which underfactor? *Intelligence*, *54*, 37–54.

Kossakowski, J. J., Epskamp, S., Kieffer, J. M., van Borkulo, C. D., Rhemtulla, M., & Borsboom, D. (2015). The application of a network approach to health-related quality of life: Introducing a new method for assessing HRQoL in healthy adults and cancer patients. Quality of life research. *Schizophrenia Bulletin* Advance Access published May 10, 2016.

Lauritzen, S. L. (1996a). *Graphical models.* Clarendon Press.

Lauritzen, S. L. (1996b). Graphical Models. *Oxford Statistical Science Series. volume 17*. New York, NY: Oxford University Press.

Must, O., & Must, A. (2014). Data from "Changes in test-taking patterns over time" concerning the Flynn Effect in Estonia. *Journal of Open Psychology Data*, *2*, e2. http://dx.doi.org/10.5334/jopd.ab.

Must, O., & Must, A. (2013). Changes in test-taking patterns over time. *Intelligence*, *41*, 780–790.

Pons, P., & Latapy, M. (2005). Computing communities in large networks using random walks. *Computer and Information Sciences-ISCIS* (pp. 284–293). Berlin Heidelberg: Springer.

Revelle, W., & Rocklin, T. (1979). Very simple structure: An alternative procedure for estimating the optimal number of interpretable factors. *Multivariate Behavioral Research*, *14*, 403–414.

Rosseel, Y (2012). lavaan: An R package for structural equation modeling. Journal of Statistical Software, 48, 1–36. URL: http://www.jstatsoft.org/v48/i02/.

Schmittmann, V. D., Cramer, A. O. J., Waldorp, L. J., Epskamp, S., Kievit, R. A., & Borsboom, D. (2013). Deconstructing the construct: A network perspective on psychological phenomena. *New Ideas in Psychology*, *31*, 43–53.

Smith Bassett, D. S., & Ed Bullmore, E. (2006). Small-world brain networks. *Neuroscientist*, *12*, 512–523.

Spearman, C. (1904). "General intelligence" objectively determined and measured. *American Journal of Psychology*, *15*, 201–293.

Tibshirani, R. (1996). Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society. Series B (Methodological)*, 267–288.

Timmerman, M. E., & Lorenzo-Seva, U. (2011). Dimensionality assessment of ordered polytomous items with parallel analysis. *Psychological Methods*, *16*, 209–220.

van Borkulo, C. D., Borsboom, D., Epskamp, S., Blanken, T. F., Boschloo, L., Schoevers, R. A., & Waldorp, L. J. (2014). A new method for constructing networks from binary data. *Scientific Reports*, *4*, 5918.

van der Maas, H. L., Dolan, C. V., Grasman, R. P., Wicherts, J. M., Huizenga, H. M., & Raijmakers, M. E. (2006). A dynamical model of general intelligence: the positive manifold of intelligence by mutualism. *Psychological review*, *113*, 842.

Velicer, W. F. (1976). Determining the number of components from the matrix of partial correlations. *Psychometrika*, *41*, 321–327.

Velicer, W. F., Eaton, C. A., & Fava, J. L. (2000). Construct explication through factor or component analysis: A review and evaluation of alternative procedures for determining the number of factors or components. In Richard D. Goffin, & Edward Helmes (Eds.), *Problems and solutions in human assessment* (pp. 41–71). New York, NY: Springer.

Žebec, M. S., Demetriou, A., & Kotrla-Topić, M. (2015). Changing expressions of general intelligence in development: A 2-wave longitudinal study from 7 to 18 years of age. *Intelligence*, *49*, 94–109.

Zwick, W. R., & Velicer, W. F. (1986). Comparison of five rules for determining the number of components to retain. *Psychological Bulletin*, *99*, 432–446.